Problema: Calculeu

$$\int_V z\sqrt{x^2+y^2}dxdydz$$

essent

$$V = \{(x,y,z) \in \mathbb{R}^3 : 0 \leq x \leq 2, 0 \leq z \leq 2, 0 \leq y \leq \sqrt{2x - x^2}\}$$

Resolem el problema utilitzant un canvi a coordenades cilíndriques

$$\left.\begin{array}{c} x = r\cos\theta \\ y = r\sin\theta \\ z = r \end{array}\right\}$$

Recordem que el jacobià d'aquest canvi és r.

$$\int_V z\sqrt{x^2+y^2}dxdydz = \int_0^{\frac{\pi}{2}}\int_0^{2\cos\theta}\int_0^2 zr^2 dzdrd\theta = \int_0^{\frac{\pi}{2}}\int_0^{2\cos\theta} 2r^2 drd\theta =$$

$$= \int_0^{\frac{\pi}{2}} \frac{2}{3}[r^3]_0^{2\cos\theta}d\theta = \int_0^{\frac{\pi}{2}} \frac{16}{3}\cos^3\theta d\theta = \frac{16}{3}\int_0^{\frac{\pi}{2}} \cos\theta(1-\sin^2\theta)d\theta =$$

$$= \frac{16}{3}\left[[\sin\theta]_0^{\frac{\pi}{2}} - \frac{1}{3}[\sin^3\theta]_0^{\frac{\pi}{2}}\right] = \frac{32}{9}$$