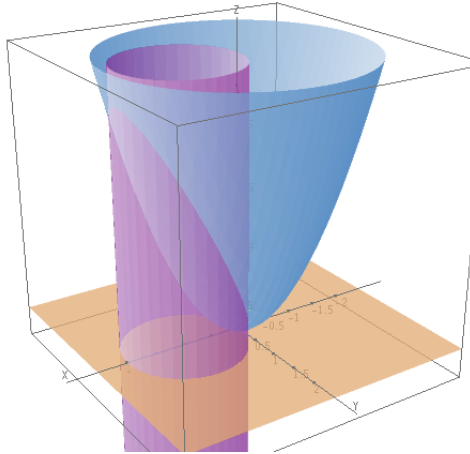

Problema: Calculeu el volum del sòlid limitat pel paraboloide $z = x^2 + y^2$ i el pla xy , dins del cilindre $x^2 + y^2 = 2x$.

Anomenem D al cercle frontera del cilindre i és $(x - 1)^2 + y^2 = 1$.



Si considerem un canvi a coordenades cilíndriques

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = z \end{cases}$$

el cercle D té la forma $r^2 = 2r \cos \theta$ o bé $r = 2 \cos \theta$.

$$D = \{(r, \theta) : -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, 0 \leq r \leq 2 \cos \theta\}$$

Així doncs, el volum que busquem és

$$\begin{aligned} V &= \iint_D (x^2 + y^2) dA = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2 \cos \theta} r^2 r dr d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\frac{r^4}{4} \right]_0^{2 \cos \theta} d\theta = \\ &= 4 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 \theta d\theta = 8 \int_0^{\frac{\pi}{2}} \cos^4 \theta d\theta = 8 \int_0^{\frac{\pi}{2}} \left(\frac{1 + \cos 2\theta}{2} \right)^2 d\theta = \\ &= 2 \int_0^{\frac{\pi}{2}} \left[1 + 2 \cos 2\theta + \frac{1}{2} (1 + \cos 4\theta) \right] d\theta = 2 \left[\frac{3}{2} \theta + \sin 2\theta + \frac{1}{8} \sin 4\theta \right]_0^{\frac{\pi}{2}} = \\ &= 2 \left(\frac{3}{2} \right) \left(\frac{\pi}{2} \right) = \frac{3\pi}{2} \end{aligned}$$
